Probabilidade 2016.1 Prova 3 08/07/16

Gabarito Resumido

① a) Seja x ponto de continuidade de X.

$$\left|\mathbb{P}[X_n \leq x] - \mathbb{P}[X \leq x]\right| \leq \mathbb{P}[X_n \leq x, X > x] + \mathbb{P}[X_n > x, X \leq x]$$

Afirm: $\mathbb{P}[X_n \leq x, X > x] \longrightarrow 0$

De fato,

$$\mathbb{P}[X_n \leq x, X > x] \leq \underbrace{\mathbb{P}[|X_n - X| \geq \varepsilon]}_{\xrightarrow[n\to\infty]{} 0} + \underbrace{\mathbb{P}[x < X \leq x+\varepsilon]}_{\leq \delta}$$

① $\downarrow n \to \infty$ 0

$\leq \delta$, pela continuidade da probabilidade

Afirm: $\mathbb{P}[X_n > x, X \leq x] \xrightarrow[n\to+\infty]{} 0$

De fato,

$$\mathbb{P}[X_n > x, X \leq x] \leq \underbrace{\mathbb{P}[|X_n - X| \geq \varepsilon]}_{\xrightarrow[n\to+\infty]{} 0} + \underbrace{\mathbb{P}[x-\varepsilon < X \leq x]}_{\leq \delta}$$

$\leq \delta$, pois x é ponto de continuidade

b) Seja $X \sim N(0,1)$

$(-1)^n X \sim N(0,1)$, logo $(-1)^n X \xrightarrow{d} N$,

mas $(-1)^n X$ não converge em probabilidade

c) $(\Leftarrow)$ vale pela letra a.

$(\Rightarrow)$ $\mathbb{P}[|X_n - 0| \geq \varepsilon] = \mathbb{P}[X_n \geq \varepsilon] + \mathbb{P}[X_n \leq -\varepsilon]$

$\xrightarrow[n\to+\infty]{} 0 + 1 - 1 = 0$.

d) $|\varphi_{c_n X_n}(t) - 1| = |\mathbb{E}[e^{itc_nX_n}] - 1|$

$\overset{\text{Jensen}}{\leq} \mathbb{E}|e^{itc_nX_n} - 1| \xrightarrow{\text{TCD}} 0$

2 a) $\varphi_{\frac{S_n}{\sqrt{n}}}(t) = \varphi_{X_1}\left(\frac{t}{\sqrt{n}}\right)^n = \left(e^{-t^2/2n}\right)^n$

$= e^{-t^2/2}$

$\Rightarrow \frac{S_n}{\sqrt{n}} \sim N(0,1)$

b) Use a letra d da 1ª questão

3) $\varphi_{\frac{S_n}{n}}(t) = \left(e^{-\frac{|t|}{n}}\right)^n = e^{-|t|}$

$\Rightarrow \frac{S_n}{n} \xrightarrow{d}$ Cauchy-Padrão.

④ a) $\mathbb{P}[M_n - \log n \leq x] =$

$$\mathbb{P}[X_1 \leq x + \log n]^n = \left(1 - e^{-x-\log n}\right)^n \xrightarrow[n\to+\infty]{}$$

$$\longrightarrow e^{(-e^{-x})}$$

b) $\mathbb{P}\left[\frac{M_n}{n^{1/\alpha}} \leq x\right] = \mathbb{P}\left[M_n \leq n^{1/\alpha} x\right]$

$$= \left(1 - \left(n^{1/\alpha} x\right)^{-\alpha}\right)^n = \left(1 - \frac{1}{x^\alpha n}\right)^n \longrightarrow e^{-(x^{-\alpha})}$$

c) $\mathbb{P}\left[n^{1/\beta} M_n \leq x\right] = \mathbb{P}\left[M_n \leq \frac{x}{n^{1/\beta}}\right] =$

$$= \left(1 - \left(\frac{|x|}{n^{1/\beta}}\right)^\beta\right)^n = \left(1 - \frac{|x|^\beta}{n}\right)^n \longrightarrow e^{-|x|^\beta}$$