Prova 2 Probabilidade 2014.2

Data 02/12/14

Gabarito resumido

(1) a) $\mathbb{P}[|n^{-X_n}| > \varepsilon] = \mathbb{P}[-X_n \log n > \log \varepsilon]$

$= \mathbb{P}\left[X_n < \frac{-\log \varepsilon}{\log n}\right] = \frac{-\log \varepsilon}{\log n}$

a) Borel-Cantelli e $\sum_n \frac{1}{\log n} = +\infty$

(2) $\mathbb{E}X_1^2 = 1$, $\mathbb{E}(X_1 - 1)^2 = 1 - 2\mathbb{E}X_1 + 1 = 2$

$$\frac{X_1^2 + \cdots + X_n^2}{(X_1-1)^2 + \cdots + (X_n-1)^2} = \frac{\frac{1}{n}(X_1^2 + \cdots + X_n^2)}{\frac{1}{n}((X_1-1)^2 + \cdots + (X_n-1)^2)}$$

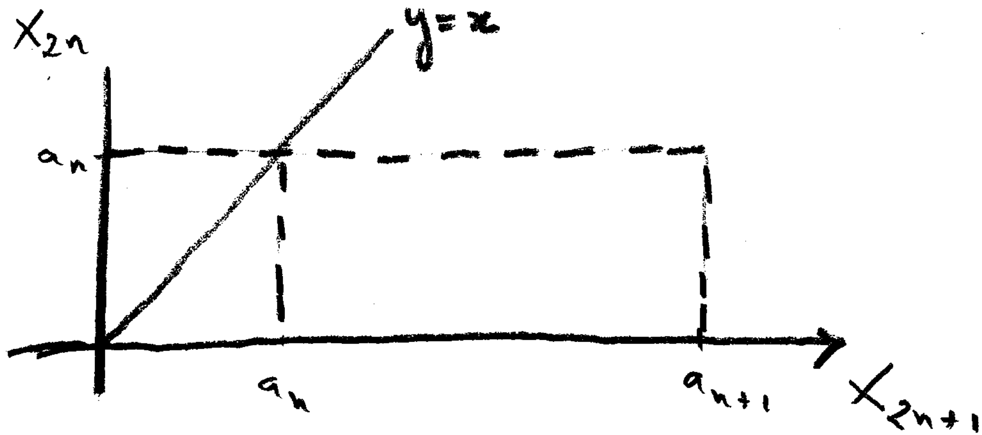
$$\xrightarrow[n\to+\infty]{q.c.} \frac{\mathbb{E}X_1^2}{\mathbb{E}(X_1-1)^2} = \frac{1}{2} \quad \text{pela L.G.N.}$$

(3) $$\frac{\frac{1}{n}\sum_{k=1}^{n} X_k}{\sqrt{\frac{1}{n}\sum_{k=1}^{n} X_k^2}} \xrightarrow[n\to+\infty]{q.c.} \frac{\mathbb{E}X_1}{\sqrt{\mathbb{E}X_1^2}} = \frac{1}{\sqrt{2}}$$

(4) $\Omega = (0,1)$, $\mathcal{A}$ = Borelianos, $\mathbb{P}$ = uniforme

$A_n = (0, \frac{1}{n})$

(5)



$$\mathbb{P}[X_{2n+1} < X_{2n}] \leq \mathbb{P}[X_{2n+1} \leq a_n] = \frac{a_n^2}{a_n \cdot a_{n+1}} =$$

$$= \frac{a_n}{a_{n+1}} = \frac{1}{n^2}$$ Logo, $\sum_n \mathbb{P}[X_{2n+1} < X_{2n}] < +\infty$

e uso Borel-Cantelli.